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An Experimental Study of Bicriteria Models for Robust Timetabling

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We conduct an experimental study for the timetabling problem in a public railway network under disruptions. We investigate three bicriteria optimization problems introduced recently that model the robustness of a timetable towards delays. We experimentally evaluated these models against various waiting time rules at stations. Our results constitute the first proofs-of-concept for these new robust timetabling approaches.

Keywords: Bicriteria optimization; robustness; timetabling problem; Pareto-optimal solutions

1. Introduction

The construction of a timetable is one of the most important phases in a public transportation network and has been extensively studied since quite some time; see e.g., [4–6, 8]. The timetabling problem asks for determining the departure and arrival times of public transportation media (e.g., trains) in order to serve, in a timely fashion, the customer demand. Typically the goal is to find an optimal solution, that is, a timetable that minimizes the overall traveling time of passengers. On the other hand, quite often there are disruptions to the normal operation of a public transportation network (e.g., due to signaling problems, maintenance work, weather conditions, accidents, etc) and this affects the timetable. As a consequence, there is a recent shift of focus towards robustness issues [2, 3], that is, providing a timetable that is robust to disruptions rather than an optimal timetable for an ideal case. Disruptions introduce an annoying overhead to passengers, but also introduce a substantial environmental overhead. Hence, robust timetables contribute towards more eco-friendly and sustainable transportation systems.

Many notions of robustness have been introduced and used for the timetabling problem (see e.g., [2]). Most of them use a given level of robustness that has to be determined beforehand. In this work, we concentrate on a new robustness approach presented recently in [7]. In that paper, the robust timetabling problem is studied as a bicriteria optimization problem, where its first objective is the objective of the nominal (undisrupted) problem, while its second objective is a suitably defined

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function measuring the robustness of the provided solution. The new approach has the advantage that one can use any robustness function as a second objective and incorporate in it any concrete measure s/he wishes to take (e.g., towards more eco-aware transportation systems). Based on common practical waiting time rules at train stations, three such robustness measures are proposed in [7], leading to three bicriteria optimization problems.

The theoretical study in [7] investigates first a fundamental case with two different trains, one arriving and the other departing at/from a given station, and a transfer activity between them, and then investigates the case of general networks. The fundamental case provides the key ingredients for studying the general case, but it is also interesting in itself as it constitutes the basic building block for other more involved cases.

In this work, we provide the first implementation and experimental evaluation of both the fundamental case and the case of general networks. The purpose of our study is to provide a proof-of-concept for the three bicriteria optimization problems in [7], modeling the robustness of the timetable towards delays against various waiting time rules at stations. Our experimental study is based on real data taken from the German railway network and constitutes the first proof-of-concept for these models.

The rest of the paper is organized as follows. In Section 2, we review the definition of the timetabling problem, as well as the robustness measures and the corresponding three optimization problems as introduced in [7]. In Section 3, we review the fundamental case studied in [7] along with the specific formulation of the optimization problems, while the approach regarding the general case is described in Section 4. In Section 5, we present the results of our experimental study, and we conclude in Section 6.

2. Preliminaries

We follow the exposition in [7]. A timetable is defined with the use of the so-called event-activity network, i.e., a directed (connected) graph $G = (E, A)$ whose node set E represents events (departures, arrivals), while its edge set $A \subseteq E \times E$ represents activities (transfer, stop, drive).

A *timetable* $\Pi \in \mathbb{R}^{|E|}$ assigns a time Π_i to each event $i \in E$. Each activity $a = (i, j)$, linking two events $i, j \in E$, is associated with a lower l_a and an upper time bound u_a that have to be respected. A timetable is called *feasible* if $\Pi_j - \Pi_i \in [l_a, u_a]$, for all $a = (i, j) \in A$.

A path P in G is a sequence of events $(i_1, \ldots, i_n)$ such that either $(i_k, i_{k+1}) \in A$ (forward activity) or $(i_{k+1}, i_k) \in A$ (backward activity), for $1 \leq k < n$. The forward (resp. backward) activities of P are denoted as P^+ (resp. P^-). A cycle C is a path with $i_1 = i_n$. The *slack times* of a timetable are defined as $s_a = \Pi_j - \Pi_i - l_a$, $\forall a \in A$. The slack time s_a is the available additional time for activity a and it is used to reduce delays. Let $s(P) = \sum_{a \in P} s_a$ be the accumulated slack along a path P.

Let $m_a = u_a - l_a$, and let w_a be the number of passengers traveling along an activity a. There are two equivalent definitions of the *timetabling* problem that aims at minimizing the overall traveling time of passengers (for details, see [7]).

- Timetabling using variables $\Pi_i, i \in E$

$$\min \mathcal{F}(\Pi) = \sum_{a=(i,j)\in A} w_a(\Pi_j - \Pi_i)$$
$$s.t.\ l_a \le \Pi_j - \Pi_i \le u_a \quad \forall a = (i,j) \in A,\ \Pi_i \in I\!N.$$

- Timetabling using variables $s_a, a \in A$

$$\min \mathcal{F}(s) = \sum_{a\in A} w_a s_a$$
$$s.t.\ 0 \le s_a \le m_a$$
$$\sum_{a\in C^+} s_a - \sum_{a\in C^-} s_a = -\sum_{a\in C^+} l_a + \sum_{a\in C^-} l_a$$

The robustness of a timetable is its sensitivity to unforseen delays. Before defining robustness formally, we need to see what happens when some delay occurs. Delays typically affect transfer activities.

When a transfer activity $a = (i,j)$ takes place, there are two possibilities: either the outgoing train waits for the delayed incoming train so that the transfer is maintained, or the train departs on time and no delay is transferred. The problem of determining which transfers must be maintained and which must not is known as the delay management problem. As it is customarily assumed, we consider this problem solved, i.e., that we are given beforehand some waiting time rules (WTRs). We consider three typical such rules.

Let i represent an arrival event of train A at a station, let j represent a departure event of another train B at the same station, let $a = (i,j)$ be a transfer activity from train A to train B, and let $a_{next} = (j,k)$ be the next driving activity of train B. Assume that train A arrives at i with a delay δ_i. The next three WTRs determine whether train B waits for train A, or departs on time.

WTR1: Train B is not allowed to have a delay at its next station. The transfer is maintained if and only if $\delta_i \le s_a + s_{a_{next}}$.

WTR2: Train B can wait n minutes, where n is fixed. The transfer is maintained if and only if $\delta_i \le s_a + n$.

WTR3: Train B is not allowed to have a delay of more than m minutes at its next station. The transfer is maintained if and only if $\delta_i \le s_a + s_{a_{next}} + m$.

We are now ready to define robustness. We assume that only one WTR is used within a public transportation system and that the timetabling problem is described using variables s_a. In particular, let a fixed WTR be given, let $s \in I\!R^{|A|}$ be a timetable, and consider a set of source-delayed events $E_{delayed}$ with delays $\delta_i \le \delta$, for all $i \in E_{delayed}$, for some given δ. Three robustness functions are defined.

- $R(s)$ measures the robustness of a timetable if all of its transfers are maintained whenever all source delays are smaller than or equal to some value R.
- $R_{no}(s, \delta)$ is the maximum number of passengers who miss a transfer if all source delays are smaller than δ.

- $R_{del}(s, \delta)$ is the maximum sum of all passengers' delays[1], if all source delays are smaller than δ.

The first definition of robustness computes the maximum source delay for which no transfer is missed. Clearly, this robustness measure needs to be maximized. The other two definitions of robustness evaluate how badly the passengers are affected by source delays. In these cases, the robustness measures need to be minimized.

The above definitions lead naturally to three bicriteria optimization problems that incorporate robustness to the timetabling problem. The first objective function is common for all models and represents the total traveling time of the passengers ($\mathcal{F}(s)$). The second objective function is one of the robustness functions just defined ($R(s), R_{no}(s, \delta), R_{del}(s, \delta)$). To summarize, the three bicriteria optimization problems are as follows:

$$\begin{array}{rll} (P) & [\min \mathcal{F}(s), \max\ R(s)] & \text{s.t. } s \text{ is a feasible timetable} \\ (P_{no}) & [\min \mathcal{F}(s), \min R_{no}(s, \delta)] & \text{s.t. } s \text{ is a feasible timetable} \\ (P_{del}) & [\min \mathcal{F}(s), \min R_{del}(s, \delta)] & \text{s.t. } s \text{ is a feasible timetable} \end{array}$$

Note that each problem can be considered for all three WTRs. In all problems we seek for *Pareto optimal* (or non-dominated) solutions, i.e., timetables s that there does not exist another timetable s' which is not worse in one of the two objectives and strictly better in the other one. A timetable is called *weak Pareto* (or weakly non-dominated) if there does not exist another timetable which is strictly better in both objectives. A Pareto solution is always weak Pareto, but the reverse is not true in general.

3. A Fundamental Case

In this section, we shall study the simplest case that models a single transfer between two trains [7]; see Fig. 1. Although simple, it is fundamental and provides the key ingredients for studying other more involved cases.

We assume that train 1 travels from station F to station M, train 2 travels from station M to station K, there is one possible transfer activity at station M, and a delay of size δ occurs at station F.

Train 1 reaches station M with a delay of $[\delta - s_1]^+$. Passengers transferring to train 2 will arrive with a delay of $[\delta - s_1 - s_2]^+$. Let us see how the transfer is maintained w.r.t. each WTR.

- Using WTR1, train 2 can wait at most s_3 minutes. Hence, the transfer is maintained if and only if $\delta - s_1 - s_2 \leq s_3 \Leftrightarrow \delta \leq s_1 + s_2 + s_3$.
- Using WTR2, train 2 can wait at most n minutes. Hence, the transfer is maintained if and only if $\delta - s_1 - s_2 \leq n \Leftrightarrow \delta \leq s_1 + s_2 + n$.
- Using WTR3, train 2 can wait at most m minutes and is not allowed to have a delay at its next station. Hence, the transfer is maintained if and only if $\delta - s_1 - s_2 \leq s_3 + m \Leftrightarrow \delta \leq s_1 + s_2 + s_3 + m$.

[1] If a passenger misses a transfer, then the delay is assumed to be T minutes, provided that the timetable is repeated after T minutes and hence the passenger can use the transfer of the next period.

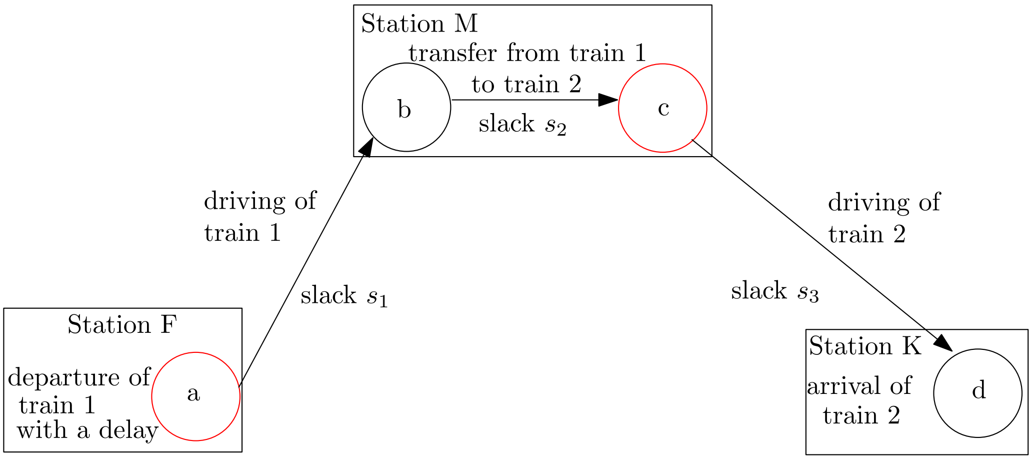

Figure 1. Fundamental case; nodes a, c (resp. b, d) represent departure (resp. arrival) events.

The above suggest that the first robustness function is defined as:

$$R(s_1, s_2, s_3) = \begin{cases} s_1 + s_2 + s_3, & \text{for WTR1} \\ s_1 + s_2 + n, & \text{for WTR2} \\ s_1 + s_2 + s_3 + m, & \text{for WTR3} \end{cases}$$

To determine the other two robustness functions, R_{no} and R_{del}, we have to take into account the number of passengers $w_{F\to M}, w_{M\to K}, w_{F\to K}$ traveling among stations F, M, and K (which is assumed given). For R_{no} only passengers traveling from F to K can miss the transfer. Hence, for any WTR, R_{no} becomes:

$$R_{no}(s, \delta) = \begin{cases} w_{F\to K} & \text{, if } \delta > R(s_1, s_2, s_3) \\ 0 & \text{, if } \delta \le R(s_1, s_2, s_3). \end{cases}$$

For R_{del}, observe that passengers from F to M gain a delay of $[\delta - s_1]^+$, passengers from F to K gain a delay of T if they miss the transfer, or they get the same delay as the passengers from M to K, which is $[\delta - s_1 - s_2 - s_3]^+$.

$$R_{del}(s, \delta) = \begin{cases} w_{F\to M}[\delta - s_1]^+ + T w_{F\to K} & \text{, if } \delta > R(s_1, s_2, s_3) \\ w_{F\to M}[\delta - s_1]^+ + (w_{F\to K} + w_{M\to K})[\delta - s_1 - s_2 - s_3]^+ & \text{, if } \delta \le R(s_1, s_2, s_3). \end{cases}$$

The first objective function $\mathcal{F}(s)$ concerns the minimization of the passengers' traveling time and is therefore common to all three problems $(P), (P_{no}), (P_{del})$.

$$\mathcal{F}(s_1, s_2, s_3) = (w_{F\to M} + w_{F\to K})s_1 + w_{F\to K}s_2 + (w_{F\to K} + w_{M\to K})s_3.$$

After determining the objective functions, we now turn to the specific problem formulations. The first problem (P) becomes:

$$\begin{aligned} &\min \mathcal{F}(s_1, s_2, s_3) \\ &\max R(s_1, s_2, s_3) \\ &\text{s.t. } 0 \le s_i \le m_i, \text{ for } i = 1, 2, 3 \end{aligned}$$

As it is proved in [7], for the solutions (s_1, s_2, s_3) of (P) it holds that $s_1 \in \{0, m_1\}$, $s_2 \in \{0, m_2\}$, and $s_3 \in \{0, m_3\}$ – the specific values depend in the used WTR. The interpretation of these solutions is that the slack should be put on the transfer and not on the driving activities.

For the second problem (P_{no}), a binary variable z is used with $z = 0$ if train 2 waits for train 1, and $z = 1$ otherwise. Then, (P_{no}) becomes:

$$\begin{aligned}
&\min \mathcal{F}(s_1, s_2, s_3)\\
&\min z w_{F\to K}\\
&s.t.\ \delta z + R(s_1, s_2, s_3) \geq \delta\\
&\qquad 0 \leq s_i \leq m_i, \text{ for } i = 1, 2, 3\\
&\qquad z \in \{0, 1\}.
\end{aligned}$$

Depending on the particular WTR used, we have the following. If the transfer is maintained, $z = 0$ and $R_{no} = 0$. If the transfer is missed, $z = 1$ and $R_{no} = w_{F\to K}$. As it is proved in [7], the Pareto solutions are of the form $(\mathcal{F}(s), 0)$, if the transfer is maintained, and of the form $(0, w_{F\to K})$, otherwise (in the latter case the traveling time of passengers is zero, because they missed the transfer and hence do not travel). The interpretation of these solutions is that either distribute no slack at all, or distribute the minimum amount of slack to maintain the transfer.

For the third problem (P_{del}), we also use the binary variable z and recall that the sum of all delays of the passengers need to be taken into account. Then, (P_{del}) becomes:

$$\begin{aligned}
&\min \mathcal{F}(s_1, s_2, s_3)\\
&\min\ (\delta - s_1) w_{F\to M} + zT w_{F\to K} + (1 - z)(w_{F\to K} + w_{M\to K})(\delta - s_1 - s_2 - s_3)\\
&s.t.\ \delta z + R(s_1, s_2, s_3) \geq \delta\\
&\qquad s_1 + s_2 + s_3 \leq \delta\\
&\qquad 0 \leq s_i \leq m_i, \quad i = 1, 2, 3\\
&\qquad z \in \{0, 1\}.
\end{aligned}$$

This model is quadratic and the linear formulation w.r.t. the three WTRs is as follows. For WTR1, if $z = 0$, then the first constraint implies that $R(s_1, s_2, s_3) \geq \delta \Leftrightarrow s_1+s_2+s_3 \geq \delta$; this in combination with the second constraint $(s_1+s_2+s_3 \leq \delta)$ gives that $\delta - s_1 - s_2 - s_3 = 0$. If $z = 1$, then the term $(1-z)(w_{F\to K}+w_{M\to K})(\delta - s_1 - s_2 - s_3)$ is zero. For WTR2 and WTR3, an auxiliary variable q is used.

In summary, (P_{del}) simplifies to the following mode for WTR1:

$$\begin{array}{ll}
\min & \mathcal{F}(s_1, s_2, s_3)\\
\min & (\delta - s_1) w_{F\to M} + zT w_{F\to K}\\
s.t. & \delta z + R(s_1, s_2, s_3) \geq \delta\\
 & s_1 + s_2 + s_3 \leq \delta\\
 & 0 \leq s_i \leq m_i,\ i = 1, 2, 3\\
 & z \in \{0, 1\}
\end{array}$$

and (P_{del}) simplifies to the following model for WTR2 and WTR3:

$$\begin{array}{ll}
\min & \mathcal{F}(s_1, s_2, s_3) \\
\min & (\delta - s_1)w_{F\to M} + zTw_{F\to K} + (w_{F\to K} + w_{M\to K})q \\
s.t. & q + \delta z + s_1 + s_2 + s_3 \geq \delta \\
& \delta z + R(s_1, s_2, s_3) \leq \delta \\
& s_1 + s_2 + s_3 \leq \delta \\
& 0 \leq s_i \leq m_i,\ i = 1, 2, 3 \\
& z \in \{0, 1\} \\
& q \geq 0
\end{array}$$

It can be proved that the two linear formulations are equivalent to the original model [7].

4. General Network

In the case of general networks, the approach suggested in [7] for studying problem (P) is to treat it as a constrained optimization problem. That is, to minimize the total passengers' traveling time (function $\mathcal{F}$), while fixing (or upper bounding) the level of robustness to a value R (where R is given beforehand). Thus, we are left with a minimization problem with one objective function ($\mathcal{F}$) and an additional constraint. In other words, problem (P)

$$(P) \quad [\min \mathcal{F}(s), \max\ R(s)] \qquad \text{s.t. } s \text{ is a feasible timetable}$$

is now transformed into the single criteria (constrained) optimization problem (SiP):

$$(SiP) \quad [\min \mathcal{F}(s)] \qquad \text{s.t. } R(s) \leq R \text{ and } s \text{ is a feasible timetable}$$

Every optimal solution of (SiP) is a weak Pareto solution of (P). To solve SiP, it remains to determine the constrains that will determine the feasible solutions.

Recall that a_{next} denotes the unique driving activity that follows transfer a, and that $E_{delayed}$ denotes the source-delayed events. Let A^{trans} be the set of all transfer activities, let A^{wait} be the set of all waiting activities, and let A^{drive} be the set of all driving activities. If we delete all transfer activities, then we obtain the subnetwork $\overline{G} = (E, A^{wait} \cup A^{drive})$. Define the set of relevant events as

$$E^{rel} = \{j \in E : \exists \text{ directed path in } \overline{G} \text{ from } i_{delayed} \in E_{delayed} \text{ to } j\}$$

Recall that $s(P)$ denotes the accumulated slack along a path P, and define

$$\overline{s}_j = \min\{s(P) : P \text{ directed path in } \overline{G} \text{ from } i_{delayed} \in E_{delayed} \text{ to } j\}$$

The computation of $\overline{s}_j$ can be easily done. Start at j and then compute backwards the shortest distance to all $i_{delayed} \in E_{delayed}$.

In [7, Theorem 3] it is proved that for a feasible timetable s, we have that $R(s) = R$ for all WTRs, if and only if for all transfers $a = (i, j) \in A^{trans}$ with $i \in E^{rel}$ it holds that:

$$\begin{array}{ll}
R \leq s_{a_{next}} + s_a + \bar{s}_i & \text{for WTR1} \\
R \leq n + s_a + \bar{s}_i & \text{for WTR2} \\
R \leq m + s_{a_{next}} + s_a + \bar{s}_i & \text{for WTR3}
\end{array}$$

The above inequalities constitute the sought constraints for (SiP).

5. Implementation and Experiments

In this section, we report on our experimental study on the aforementioned models both for the fundamental and the general case. Our implementations were conducted using `C++` (compiler gcc version 4.1.2) and CPLEX 10.1. For our experiments we used real-world data from the German Railways [1], regarding lines passing from the main train stations of the cities shown in Table 1. The particular lines used for each case will be specified in the rest of the section.

Station	Short Name
Belin	B
Dresden	DD
Hannover	H
Leipzig	L
Kaiserslautern	KA
Fulda	FD
Mannheim	MA
Frankfurt	F
Basel	BS
Zürich	ZH

Table 1. Main train stations and their short name.

In the rest of the paper, we shall refer to the stations using their short names (see Table 1), and use the notation $w_{X \to Y}$ to denote the number of passengers travelling from station X to station Y.

5.1. *Experimental Study of the Fundamental Case*

For the fundamental case, we considered the case of passengers traveling from Frankfurt (F) to Kaiserslautern (KA) with an intermediate transfer at Mannheim (MA), using ICE (intercity express) and IC (intercity) trains. We used the timetable of August 2010 (for one particular day) and considered the capacity of ICE trains to 415 passengers and that of IC trains to 1000 passengers. The overall 23 routes and the train connections are shown in Table 2. The number of passengers $w_{F \to MA}$, $w_{F \to KA}$, $w_{MA \to KA}$ traveling in each activity (F→MA, F→KA, and MA→KA, resp.) was a random value in $[1, 415]$ if at least one ICE train was involved, and a a random value in $[1, 1000]$ otherwise. We also assumed that when a transfer is made from train 1 to train 2, the latter train had the same or bigger capacity than the former. Each bicriterion optimization problem had to be solved for every WTR and there are 23 routes to be considered in each case, leading to a vast amount of results. We report here on selected results representing the most interesting cases (concentrating on the strictest rule WRT1 and on WRT2).

We start with (P) using WTR1. The data used and the computed slack times are shown in Table 3 and concern routes 5-10 of Table 2. The results are reported in Fig. 2, with the horizontal axis representing function R (that needs to be maximized) and the vertical axis representing function $\mathcal{F}$ (that needs to be minimized). The legend in Fig. 2 shows the values of $\mathcal{F}$ and R. The results confirm the theoretical solutions discussed in Section 3. Observe that (see Table 3) in every triple (s_1, s_2, s_3) the slack variables get either their lower bound or their upper bound. In all triples except the first one, variables s_2 get their maximum slack, which confirms the theoretical observation that slack should be put on transfer activities.

Pareto solutions for (P_{no}) using WTR2 are shown in Table 4. The data shown are: n (number of minutes regarding WTR2), δ (source delay in minutes), $w_{F \to MA}, w_{F \to KA}, w_{MA \to KA}$ (number of passengers for the corresponding activi-

Routes (id's)	Departure F	Arrival MA	Departure MA	Arrival KA
1	06:50	07:28	07:56	08:59
2	07:50	08:28	08:56	09:59
3	09:13	10:20	10:26	11:28
4	10:05	10.42	10:55	11:59
5	10:50	11:28	11:56	12:59
6	11:13	12:20	12:26	13:28
7	12:05	12:42	12:56	13:59
8	13:13	14:20	14:26	15:28
9	14:05	14:42	14:56	15:59
10	14:50	15:28	15:56	16:59
11	15:13	16:20	16:26	17:28
12	16:05	16:42	16:56	17:59
13	17:13	18:20	18:26	19:28
14	18:05	18:42	18:56	19:59
15	18:50	19:28	19:56	20:59
16	19:13	20:20	20:26	21:26
17	20:05	20:42	20:50	21:33
18	21:05	21:53	22:08	23:14
19	21:50	22:28	22:40	23:45
20	22:05	22:42	23:12	00:18
21	23:00	23:41	00:01	01:43
22	23:13	00:20	00:31	02:07
23	00:18	01:10	01:16	02:19

Table 2. Timetable for a single day (August 2010) for ICE and IC trains.

Input Data							Solutions
Id	$w_{F\to MA}$	$w_{F\to KA}$	$w_{MA\to KA}$	s_1	s_2	s_3	(s_1, s_2, s_3)
5	93	77	105	[0,7]	[0,28]	[0,6]	{(0,0,0),(0,28,0),(7,28,0),(7,28,6)}
6	55	13	55	[0,3]	[0,6]	[0,1]	{(0,0,0),(0,6,0),(3,6,1)}
7	158	242	307	[0,3]	[0,14]	[0,5]	{(0,0,0),(0,14,0),(3,14,0),(3,14,5)}
8	80	19	235	[0,2]	[0,6]	[0,7]	{(0,0,0),(0,6,0),(2,6,0),(2,6,7)}
9	183	150	457	[0,8]	[0,14]	[0,3]	{(0,0,0),(0,14,0),(8,14,0),(8,14,3)}
10	539	107	350	[0,7]	[0,28]	[0,8]	{(0,0,0),(0,28,0),(0,28,8),(7,28,8)}

Table 3. Input data and computed slack times for (P) for routes 5-10.

ties), the values of $\mathcal{F}$ and R when the transfer is missed ($z = 1$), and when is maintained ($z = 0$). In the latter case, the slack times are also shown with their upper bounds. The data correspond to routes 20-22. There are only two non-dominated solutions and hence the results confirm the theoretical solutions discussed in Section 3. In routes 20-22 the time provided by WTR2 reduces the delay δ but does not absorb it, so either the minimum amount of slack is given in order to maintain the transfer or there is no slack at all and the transfer is missed.

						$z=0$								$z=1$	
Id	n	δ	$w_{F\to MA}$	$w_{F\to KA}$	$w_{MA\to KA}$	$\mathcal{F}$	R	s_1	m_1	s_2	m_2	s_3	m_3	$\mathcal{F}$	R
20	8	42	100	50	210	1800	0	2	3	30	30	2	10	0	50
21	1	28	88	77	96	2695	0	7	7	20	20	0	9	0	77
22	11	13	400	219	100	438	0	0	4	2	11	0	4	0	219

Table 4. Input data and Pareto solutions for P_{no} regarding routes 20-22.

Finally, we report results for (P_{del}) using WTR1. The data used and the computed slack times are shown in Table 5, and concern routes 11-16. The results are reported in Fig. 3, with the horizontal axis representing function R and the vertical axis representing function $\mathcal{F}$ (both need to be minimized). Note that there are solutions with $\mathcal{F} = 0$. This means that the transfer was missed and the sum of all passengers' delays is R. When the transfer is maintained, the values of R are lower, but $\mathcal{F}$ increases as we are dealing with two conflicting functions. The results suggest that we should distribute slack equal to the delay when the transfer is maintained in order to absorb it. When the transfer is missed, either distribute no slack at all or distribute an amount of slack to minimize the maximum sum of all passengers' delays.

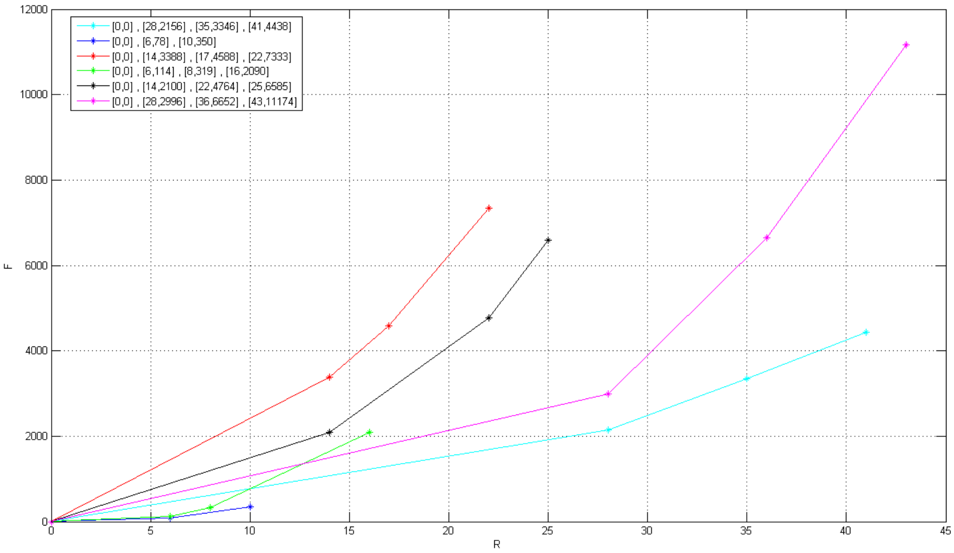

Figure 2. Pareto solutions for (P) concerning routes 5-10 using WTR1.

Input Data							
Id	δ	$w_{F\to MA}$	$w_{F\to KA}$	$w_{MA\to KA}$	s_1	s_2	s_3
11	5	237	73	40	[0,2]	[0,6]	[0,3]
12	19	607	302	41	[0,3]	[0,14]	[0,3]
13	7	78	108	61	[0,2]	[0,6]	[0,2]
14	18	109	350	200	[0,10]	[0,14]	[0,10]
15	25	300	309	102	[0,10]	[0,28]	[0,10]
16	10	50	80	152	[0,3]	[0,6]	[0,7]

Solutions	
Id	(s_1, s_2, s_3, z)
11	{(2,14,3,0),(3,14,2,0),(0,0,0,1),(3,0,0,1)}
12	{(0,5,0,0),(2,3,0,0),(0,0,0,1),(2,0,0,1)}
13	{(0,6,1,0),(1,6,0,0),(2,5,0,0),(0,0,0,1),(2,0,0,1)}
14	{(4,14,0,0),(10,8,0,0),(0,0,0,1),(2,0,0,1)}
15	{(0,25,0,0),(10,15,0,0),(0,0,0,1),(10,0,0,1)}
16	{(3,6,1,0),(0,0,0,1),(3,0,0,1)}

Table 5. Input data and computed slack times for (P_{del}) regarding routes 11-16.

5.2. *Experimental Study of the General Case*

For the general case, we conducted experiments with lines passing through the cities shown in Table 1. In particular, we considered two major types of routes, from Zürich (ZH) to Berlin (B), and from Kaiserslautern (KA) to Dresden (DD) with one or more intermediate transfer activities to the other cities. The specific connections along these major routes between the various cities are shown in Figure 4.

For the experiments, we also used real-world data from German Railways [1]. Tables 6 and 7 show the timetable regarding these two major routes and their intermediate transfers, for a single week day. There are two rows for every route. The first shows the arrival time at the station and the second shows the departure time from the station. Stations in which the train does not stop are marked with a dash (-). We used the timetable of December 2011 (one particular day). Again train types are Intercity (IC) and Intercity Express (ICE). We make the same assumptions regarding the capacity of the trains and the intermediate transfers as those of Section 5.1.

In Tables 8, 9, and 10 the input and output data regarding each route are shown. The first column is the route id, the second column shows the number of passengers for each route and slack times with their lower and upper values. The third column

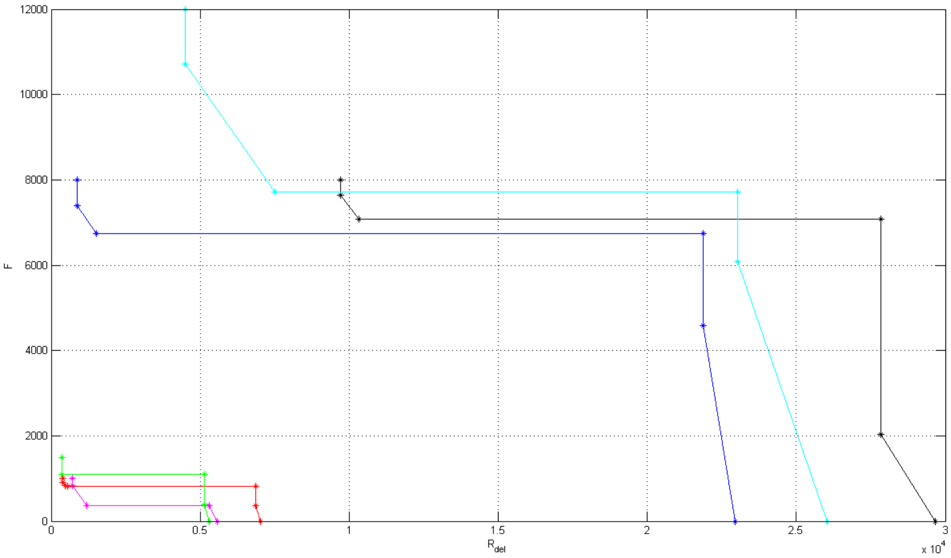

Figure 3. Weak Pareto solutions for (P_{del}) for the routes 11-16 using WTR1.

shows the value of $\mathcal{F}$ (passengers' total traveling time), R (robustness function fixed beforehand) and a tuple with the exact values for the slack times participating in each route. All three WTRs are taken into consideration and the values of n, m regarding WTR2,WTR3 are also shown.

In Tables 8, 9, and 10 there are routes with one, two or three transfer activities. The main observation of the results is that for all WTRs transfer activities often get their maximum values. Furthermore, driving activities get low values and are often equal to zero. This confirms the theoretical analysis and the observation that the slack should be put on transfer activities in order to maintain them. Regarding WTR2 and WTR3 the extra time available (n and m minutes respectively) reduces significantly the passengers' total traveling time (function $\mathcal{F}$). This is the case because in these WTRs slack variables get lower values than in WTR1, since n, m appear in the constraints. The values of n and m were chosen randomly in the interval [3,10].

A specific effort was devoted in order to select the maximum value of R for which a feasible solution exists, since R lower bounds the overall slack times. We determined this value as follows. Let R_{max} denote the maximum value of R for which there is a feasible solution. Consider a route with N transfer activities and let us select the first waiting time rule (WTR1). For every transfer activity, the constraints that have to be respected are of the form:

$$\begin{aligned}
s_{1i_1} + s_{1i_2} + \cdots + s_{1i_{K_1}} &\geq R \qquad \text{for transfer } 1 \\
s_{2j_1} + s_{2j_2} + \cdots + s_{2j_{K_2}} &\geq R \qquad \text{for transfer } 2 \\
&\qquad\qquad\qquad\quad \ldots \\
s_{Nl_1} + s_{Nl_2} + \cdots + s_{Nl_{K_N}} &\geq R \qquad \text{for transfer } N
\end{aligned}$$

where $s_{\rho\lambda_f}$, $1 \leq \rho \leq N$ and $1 \leq \lambda_f \leq K_\rho$, are the slack times participating in each transfer and driving activity. From the form of the constraint inequalities, it is obvious that in order to get feasible solutions R must be in the interval $[0, R_{max}]$.

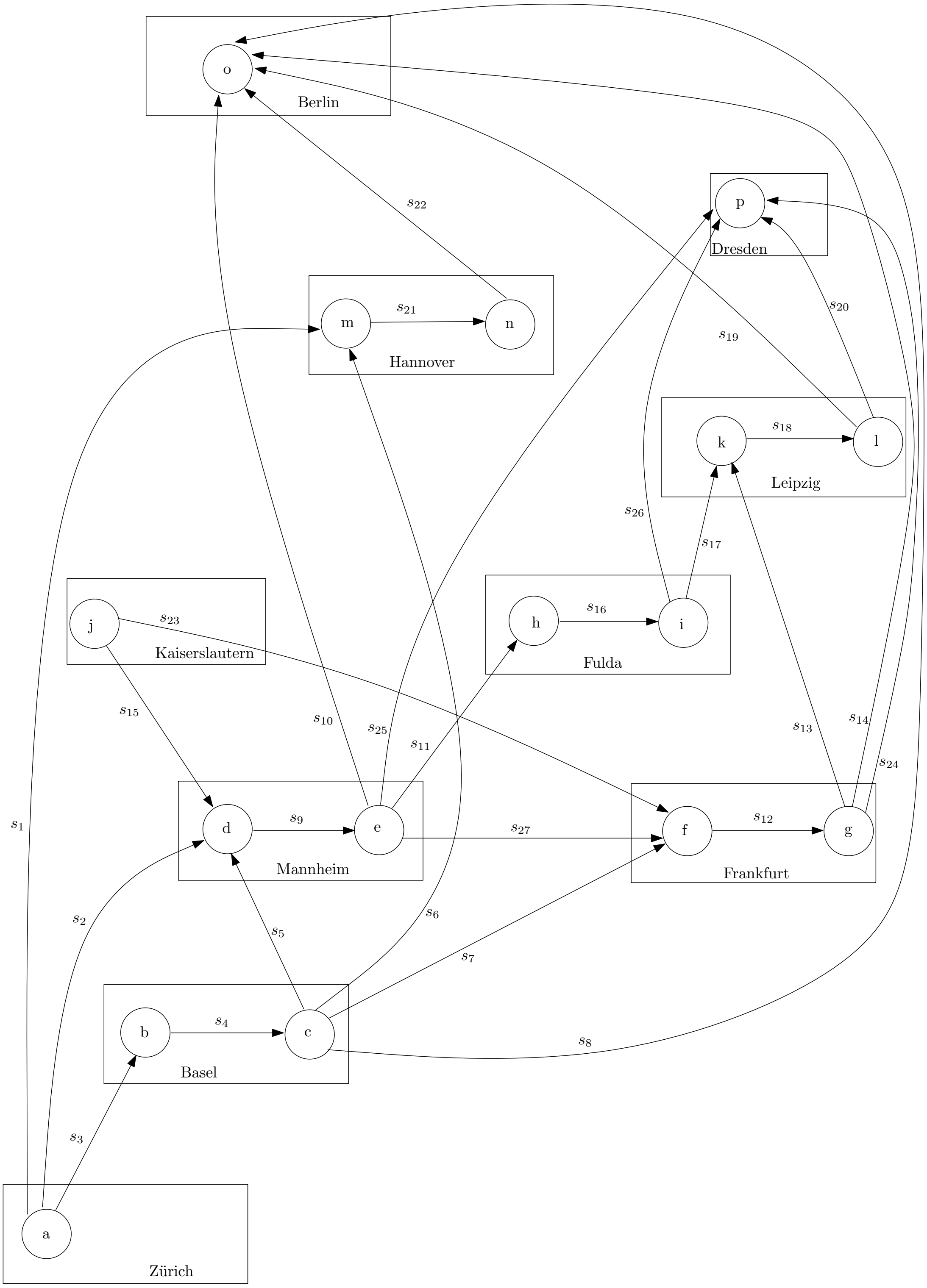

Figure 4. A general network with 10 stations. Nodes represent events (departures,arrivals) and edges represent activities(drive,transfer). There are two main routes: from Zürich to Berlin and from Kaiserslautern to Dresden

Our goal is to determine R_{max} and try to enforce R towards its R_{max} value. From Tables 6 and 7, we obtain the exact slack times for every transfer activity. For the slack times of each driving activity we have to make assumptions. Note that R_{max} depends of the sum of all slack times participating in each route. If we assume that the slack time for each driving activity is 10 minutes (a common case in rail

Routes (id's)	Stations ZH	H	BS	MA	F	L	B
1. ZH - H - B	-	12:17	-	-	-	-	14:12
	06:00	12:31	-	-	-	-	-
2. ZH - BS - F - L - B	-	-	07:53	-	11:08	14:46	16:06
	07:00	-	08:12	-	11:19	14:51	-
3. ZH - BS - B	-	-	07:53	-	-	-	15:25
	07:00	-	08:12	-	-	-	-
4. ZH - H - B	-	14:17	-	-	-	-	16:12
	08:00	14:31	-	-	-	-	-
5. ZH - BS - B	-	-	09:53	-	-	-	17:25
	09:00	-	10:12	-	-	-	-
6. ZH - BS - F - L - B	-	-	09:53	-	13:08	16:46	18:05
	09:00	-	10:12	-	13:19	16:51	-
7. ZH - H - B	-	16:17	-	-	-	-	18:11
	10:00	16:31	-	-	-	-	-
8. ZH - BS - B	-	-	11:53	-	-	-	19.25
	11:00	-	12:12	-	-	-	-
9. ZH - BS - F - L - B	-	-	11:53	-	15:08	18:46	20:07
	11:00	-	12:12	-	15:20	18:51	
10. ZH - H - B	-	18:17	-	-	-	-	20:11
	12:00	18:31	-	-	-	-	-
11. ZH - BS - B	-	-	13:53	-	-	-	21:25
	13:00	-	14:12	-	-	-	-
12. ZH - BS - F - B	-	-	14:53	-	17:53	-	21:48
	14:00	-	15:04	-	18:13	-	-
13. ZH - BS - H - B	-	20:17	14:53	-	-	-	22:21
	14:00	20:31	15:04	-	-	-	-
14. ZH - BS - B	-	-	15:53	-	-	-	23:25
	15:00	-	16:12	-	-	-	-
15. ZH - BS - MA - B	-	-	16:53	19:14	-	-	00:35
	16:00	-	17:04	19:31	-	-	-
16. ZH - MA - B	-	-	-	20:22	-	-	04:26
	17:00	-	-	20:35	-	-	-
17. ZH - BS - B	-	-	20:53	-	-	-	07:19
	20:00	-	21:12	-	-	-	-
18. ZH - BS - H - B	-	06:30	21:53	-	-	-	09:10
	21:00	07:31	22:12	-	-	-	-
19. ZH - BS - B	-	-	22:53	-	-	-	09:19
	22:00	-	23:12	-	-	-	-

Table 6. Timetable for a single day (December 2011) for ICE and IC trains. Stations involved are: Zürich (ZH), Hannover (H), Basel (BS), Frankfurt (F), Leipzig (L), Mannheim (MA), Berlin (B)

networks), the slack time for the transfer activity is 20 minutes, and that there are 3 slack times in a route, then $R_{max} = 40$. Hence, in order to get feasible solutions we must choose a value for R that belongs in the interval [0,40]. For all routes a value of R was chosen carefully in order to get feasible solutions. In some routes (e.g., Route 1), R was set to its highest value ($R = R_{max}$), but this may not be true for every route.

6. Conclusions

In this paper we conducted an experimental study of three bicriteria models proposed in [7] for the robust timetabling problem in a public railway network under disruptions. We conducted experiments regarding both the general case as well as a simple but fundamental case, under three waiting time rules (WTRs). Our experimental results confirm the theoretical analysis in [7], constitute the first proofs-of-concept for these models, and suggest that a bicriteria optimization modeling

Routes (id's)	Stations					
	KA	MA	F	FD	L	DD
20. KA - F - DD	-	-	10:58	-	-	16:04
	09:37	-	11:19	-	-	-
21. KA - MA - FD - DD	-	11:12	-	13:09	-	17:28
	10:27	11:31	-	13:14	-	-
22. KA - F - DD	-	-	12:58	-	-	18:04
	11:35	-	13:19	-	-	
23. KA - MA - FD - L - DD	-	13:03	-	15:09	18:15	19:28
	11:58	13:31	-	15:14	18:24	-
24. KA - MA - F - DD	-	14:21	15:08	-	-	20:04
	13:36	14:31	15:20	-	-	
25. KA - MA - FD - DD	-	15:03	-	17:09	-	21:42
	13:58	15:31	-	17:14	-	-
26. KA - F - DD	-	-	16:58	-	-	22:06
	15:37	-	17:20	-	-	-
27. KA - MA - F - DD	-	18:20	19:08	-	-	00:16
	17:30	18:31	19:19	-	-	-
28. KA - MA - F - DD	-	20:17	22:05	-	-	07:00
	19:37	20:35	22:28	-	-	-
29. KA - MA - DD	-	23:34	-	-	-	08:04
	22:32	23:59	-	-	-	-
30. KA - MA - F - L - DD	-	01:08	05:35	-	10:16	11:28
	00:06	04:40	06:18	-	10:24	-

Table 7. Timetable for a single day (December 2011) for ICE and IC trains. Stations involved are: Kaiserslautern (K), Fulda (FD), Frankfurt (F), Leipzig (L), Mannheim (MA), Dresden (DD)

of the robust timetabling problem is a promising direction. An interesting open issue is to investigate the applicability of the bicretira modeling to other robust optimization problems.

References

[1] `http://www.bahn.de/`.
[2] R. Ahuja, R. Moehring, and C. Zaroliagis, *Robust and Online Large-Scale Optimization*, Springer, 2009.
[3] ARRIVAL, *Project*, `http://arrival.cti.gr`.
[4] X. Cai and C.J. Goh, *A fast heuristic for the train scheduling problem*, Computers and Operation Research 21 (1994), pp. 499–510.
[5] K. Nachtigall, *Exact solution methods for periodic programs*, Tech. Rep. 14/93, Hildesheimer Informatik-Berichte, 1993.
[6] R.T. Rockafellar, *Network flows and monotropic optimization*, John Wiley & Sons, Inc., 1984.
[7] A. Schoebel and A. Kratz, *A Bicriteria Approach for Robust Timetabling*, in *Robust and Online Large-Scale Optimization*, Springer, 2009, pp. 119–144.
[8] B. Szpigel, *Optimal Train Scheduling on a Single Track Railway*, in *Proceedings of IFORS Conference on Operational Research'72*, 1973, pp. 343–352.

Id	Num. of Passengers - Slack Times		WTR1	WTR2	WTR3
1.	$w_{ZH\to B} = 64$ $w_{H\to B} = 110$ $w_{ZH\to H} = 110$	$0 \le s_1 \le 9$ $0 \le s_{21} \le 14$ $0 \le s_{22} \le 12$	$\mathcal{F} = 3368$ $R = 35$ $[9, 14, 12]$	$\mathcal{F} = 1456$ $R = 25$ $n = 7$ $[4, 14, 0]$	$\mathcal{F} = 2808$ $R = 35$ $m = 4$ $[5, 14, 12]$
2.	$w_{ZH\to BS} = 184$ $w_{ZH\to B} = 117$ $w_{BS\to F} = 184$ $w_{BS\to B} = 66$ $w_{L\to B} = 39$ $w_{ZH\to F} = 123$ $w_{ZH\to L} = 45$ $w_{BS\to L} = 92$ $w_{F\to B} = 106$	$0 \le s_3 \le 7$ $0 \le s_4 \le 19$ $0 \le s_7 \le 9$ $0 \le s_{12} \le 11$ $0 \le s_{13} \le 5$ $0 \le s_{18} \le 5$ $0 \le s_{19} \le 8$	$\mathcal{F} = 10408$ $R = 22$ $[7, 5, 10, 0, 5, 0, 0]$	$\mathcal{F} = 9703$ $R = 25$ $n = 8$ $[7, 10, 10, 0, 0, 0, 0]$	$\mathcal{F} = 9697$ $R = 24$ $m = 3$ $[7, 4, 10, 0, 4, 0, 0]$
3.	$w_{ZH\to B} = 84$ $w_{BS\to B} = 105$ $w_{Z\to BS} = 71$	$0 \le s_3 \le 5$ $0 \le s_4 \le 19$ $0 \le s_8 \le 15$	$\mathcal{F} = 5017$ $R = 38$ $[5, 19, 14]$	$\mathcal{F} = 2371$ $R = 33$ $n = 9$ $[5, 19, 0]$	$\mathcal{F} = 3505$ $R = 38$ $m = 8$ $[5, 19, 6]$
4.	$w_{ZH\to B} = 74$ $w_{H\to B} = 67$ $w_{ZH\to H} = 76$	$0 \le s_1 \le 10$ $0 \le s_{14} \le 14$ $0 \le s_{22} \le 16$	$\mathcal{F} = 4042$ $R = 35$ $[5, 14, 16]$	$\mathcal{F} = 2086$ $R = 25$ $n = 4$ $[7, 14, 0]$	$\mathcal{F} = 3292$ $R = 35$ $m = 5$ $[0, 14, 16]$
5.	$w_{ZH\to B} = 97$ $w_{BS\to B} = 62$ $w_{Z\to BS} = 76$	$0 \le s_3 \le 8$ $0 \le s_4 \le 19$ $0 \le s_8 \le 11$	$\mathcal{F} = 4803$ $R = 37$ $[7, 19, 11]$	$\mathcal{F} = 3054$ $R = 30$ $n = 4$ $[7, 19, 0]$	$\mathcal{F} = 4111$ $R = 37$ $m = 4$ $[3, 19, 11]$
6.	$w_{ZH\to BS} = 207$ $w_{ZH\to L} = 72$ $w_{BS\to B} = 102$ $w_{ZH\to B} = 138$ $w_{L\to B} = 143$ $w_{ZH\to F} = 99$ $w_{BS\to F} = 113$ $w_{BS\to L} = 88$ $w_{F\to B} = 53$	$0 \le s_3 \le 5$ $0 \le s_4 \le 19$ $0 \le s_7 \le 8$ $0 \le s_{12} \le 11$ $0 \le s_{13} \le 9$ $0 \le s_{18} \le 5$ $0 \le s_{19} \le 8$	$\mathcal{F} = 15650$ $R = 26$ $[7, 19, 10, 4, 5, 4, 0]$	$\mathcal{F} = 13928$ $R = 26$ $n = 6$ $[7, 13, 10, 3, 0, 3, 0]$	$\mathcal{F} = 12644$ $R = 28$ $m = 5$ $[7, 6, 10, 1, 5, 1, 0]$
7.	$w_{ZH\to B} = 114$ $w_{H\to B} = 56$ $w_{ZH\to H} = 83$	$0 \le s_1 \le 8$ $0 \le s_{21} \le 14$ $0 \le s_{22} \le 10$	$\mathcal{F} = 4675$ $R = 31$ $[7, 14, 10]$	$\mathcal{F} = 3172$ $R = 28$ $n = 6$ $[8, 14, 0]$	$\mathcal{F} = 4048$ $R = 31$ $m = 3$ $[4, 14, 10]$
8.	$w_{ZH\to B} = 252$ $w_{BS\to B} = 89$ $w_{ZH\to BS} = 169$	$0 \le s_3 \le 10$ $0 \le s_4 \le 19$ $0 \le s_8 \le 13$	$\mathcal{F} = 10905$ $R = 36$ $[4, 19, 13]$	$\mathcal{F} = 8577$ $R = 33$ $n = 5$ $[9, 19, 0]$	$\mathcal{F} = 8198$ $R = 36$ $m = 7$ $[0, 19, 10]$
9.	$w_{ZH\to BS} = 241$ $w_{ZH\to L} = 97$ $w_{BS\to B} = 42$ $w_{F\to B} = 70$ $w_{L\to B} = 74$ $w_{ZH\to F} = 78$ $w_{BS\to F} = 90$ $w_{ZH\to B} = 116$ $w_{BS\to L} = 122$	$0 \le s_3 \le 6$ $0 \le s_4 \le 19$ $0 \le s_7 \le 7$ $0 \le s_{12} \le 12$ $0 \le s_{13} \le 10$ $0 \le s_{18} \le 5$ $0 \le s_{19} \le 8$	$\mathcal{F} = 18689$ $R = 29$ $[6, 16, 7, 6, 10, 5, 1]$	$\mathcal{F} = 19066$ $R = 31$ $n = 7$ $[6, 18, 7, 11, 6, 5, 0]$	$\mathcal{F} = 14135$ $R = 30$ $m = 6$ $[6, 11, 7, 1, 10, 1, 0]$
10.	$w_{ZH\to B} = 164$ $w_{H\to B} = 87$ $w_{ZH\to H} = 103$	$0 \le s_1 \le 15$ $0 \le s_{21} \le 14$ $0 \le s_{22} \le 14$	$\mathcal{F} = 8480$ $R = 38$ $[10, 14, 14]$	$\mathcal{F} = 6301$ $R = 36$ $n = 7$ $[15, 14, 0]$	$\mathcal{F} = 6611$ $R = 38$ $m = 7$ $[3, 14, 14]$

Table 8. Input/Output data regarding routes 1-10.

Id	Num. of Passengers - Slack Times		WTR1	WTR2	WTR3
11.	$w_{ZH\to B} = 206$	$0 \leq s_3 \leq 12$	$\mathcal{F} = 10645$	$\mathcal{F} = 7864$	$\mathcal{F} = 8670$
	$w_{BS\to B} = 193$	$0 \leq s_4 \leq 19$	$R = 38$	$R = 38$	$R = 38$
	$w_{ZH\to BS} = 189$	$0 \leq s_8 \leq 9$		$n = 9$	$m = 5$
			$[10, 19, 9]$	$[10, 19, 0]$	$[5, 19, 9]$
12.	$w_{ZH\to BS} = 110$	$0 \leq s_3 \leq 7$	$\mathcal{F} = 7111$	$\mathcal{F} = 5969$	$\mathcal{F} = 8787$
	$w_{BS\to B} = 110$	$0 \leq s_4 \leq 11$	$R = 20$	$R = 25$	$R = 32$
	$w_{ZH\to B} = 105$	$0 \leq s_{14} \leq 5$		$n = 9$	$m = 8$
	$w_{ZH\to F} = 99$	$0 \leq s_7 \leq 6$			
	$w_{F\to B} = 97$	$0 \leq s_{12} \leq 20$	$[7, 7, 6, 7, 0]$	$[7, 9, 0, 9, 0]$	$[7, 11, 6, 11, 0]$
	$w_{BS\to F} = 126$				
13.	$w_{ZH\to BS} = 183$	$0 \leq s_3 \leq 5$	$\mathcal{F} = 7547$	$\mathcal{F} = 5835$	$\mathcal{F} = 5324$
	$w_{H\to B} = 129$	$0 \leq s_4 \leq 11$	$R = 23$	$R = 22$	$R = 22$
	$w_{ZH\to B} = 85$	$0 \leq s_6 \leq 9$		$n = 6$	$m = 5$
	$w_{ZH\to H} = 118$	$0 \leq s_{21} \leq 14$			
	$w_{BS\to B} = 67$	$0 \leq s_{22} \leq 6$	$[3, 11, 9, 11, 0]$	$[5, 11, 0, 11, 0]$	$[0, 8, 9, 8, 0]$
	$w_{BS\to H} = 73$				
14.	$w_{ZH\to B} = 198$	$0 \leq s_3 \leq 13$	$\mathcal{F} = 10162$	$\mathcal{F} = 7656$	$\mathcal{F} = 6622$
	$w_{BS\to B} = 88$	$0 \leq s_4 \leq 19$	$R = 39$	$R = 38$	$R = 39$
	$w_{ZH\to BS} = 156$	$0 \leq s_8 \leq 10$		$n = 9$	$m = 10$
			$[10, 19, 10]$	$[11, 19, 0]$	$[0, 19, 10]$
15.	$w_{ZH\to BS} = 175$	$0 \leq s_3 \leq 7$	$\mathcal{F} = 8795$	$\mathcal{F} = 5643$	$\mathcal{F} = 7767$
	$w_{BS\to B} = 139$	$0 \leq s_4 \leq 11$	$R = 20$	$R = 20$	$R = 23$
	$w_{ZH\to B} = 111$	$0 \leq s_5 \leq 5$		$n = 8$	$m = 5$
	$w_{ZH\to MA} = 153$	$0 \leq s_9 \leq 17$			
	$w_{MA\to B} = 73$	$0 \leq s_{10} \leq 4$	$[7, 8, 5, 8, 0]$	$[7, 5, 0, 5, 0]$	$[7, 6, 5, 6, 0]$
	$w_{BS\to MA} = 58$				
16.	$w_{ZH\to B} = 153$	$0 \leq s_2 \leq 7$	$\mathcal{F} = 5745$	$\mathcal{F} = 4407$	$\mathcal{F} = 4657$
	$w_{MA\to B} = 119$	$0 \leq s_9 \leq 19$	$R = 30$	$R = 30$	$R = 33$
	$w_{ZH\to MA} = 97$	$0 \leq s_{25} \leq 5$		$n = 5$	$m = 7$
			$[7, 19, 4]$	$[6, 19, 0]$	$[7, 19, 0]$
17.	$w_{ZH\to B} = 236$	$0 \leq s_3 \leq 5$	$\mathcal{F} = 9009$	$\mathcal{F} = 5976$	$\mathcal{F} = 6506$
	$w_{BS\to B} = 101$	$0 \leq s_4 \leq 19$	$R = 32$	$R = 32$	$R = 32$
	$w_{ZH\to BS} = 137$	$0 \leq s_8 \leq 9$		$n = 9$	$m = 7$
			$[4, 19, 9]$	$[4, 19, 0]$	$[0, 19, 6]$
18.	$w_{ZH\to BS} = 207$	$0 \leq s_3 \leq 11$	$\mathcal{F} = 16503$	$\mathcal{F} = 14138$	$\mathcal{F} = 13958$
	$w_{BS\to B} = 92$	$0 \leq s_4 \leq 19$	$R = 33$	$R = 35$	$R = 35$
	$w_{ZH\to B} = 109$	$0 \leq s_6 \leq 5$		$n = 7$	$m = 7$
	$w_{ZH\to H} = 193$	$0 \leq s_{21} \leq 61$			
	$w_{H\to B} = 153$	$0 \leq s_{22} \leq 4$	$[9, 19, 5, 19, 0]$	$[9, 19, 0, 19, 0]$	$[4, 19, 5, 19, 0]$
	$w_{BS\to H} = 171$				
19.	$w_{ZH-B} = 177$	$0 \leq s_3 \leq 6$	$\mathcal{F} = 7258$	$\mathcal{F} = 4475$	$\mathcal{F} = 5893$
	$w_{H-B} = 76$	$0 \leq s_4 \leq 19$	$R = 34$	$R = 28$	$R = 35$
	$w_{ZH-BS} = 101$	$0 \leq s_8 \leq 11$		$n = 5$	$m = 6$
			$[4, 19, 11]$	$[4, 19, 0]$	$[0, 19, 10]$
20.	$w_{KA-DD} = 150$	$0 \leq s_{12} \leq 21$	$\mathcal{F} = 7068$	$\mathcal{F} = 4836$	$\mathcal{F} = 4142$
	$w_{F-DD} = 98$	$0 \leq s_{23} \leq 8$	$R = 36$	$R = 35$	$R = 35$
	$w_{KA-F} = 131$	$0 \leq s_{24} \leq 9$		$n = 8$	$m = 10$
			$[6, 21, 9]$	$[6, 21, 0]$	$[0, 21, 4]$

Table 9. Input/Output data regarding routes 11-20.

Id	Num. of Passengers - Slack Times		WTR1	WTR2	WTR3
21.	$w_{KA\to MA} = 135$	$0 \le s_{15} \le 5$	$\mathcal{F} = 12966$	$\mathcal{F} = 11841$	$\mathcal{F} = 10350$
	$w_{MA\to DD} = 92$	$0 \le s_9 \le 19$	$R = 25$	$R = 25$	$R = 25$
	$w_{KA\to DD} = 159$	$0 \le s_{26} \le 6$		$n = 7$	$m = 7$
	$w_{KA\to FD} = 152$				
	$w_{FD\to DD} = 107$	$0 \le s_{11} \le 9$	$[9, 19, 5, 19, 0]$	$[9, 19, 0, 19, 0]$	$[4, 19, 5, 19, 0]$
	$w_{MA\to FD} = 142$ $0 \le s_{16} \le 5$				
22.	$w_{KA\to DD} = 178$	$0 \le s_{12} \le 21$	$\mathcal{F} = 6663$	$\mathcal{F} = 4599$	$\mathcal{F} = 5471$
	$w_{F\to DD} = 120$	$0 \le s_{23} \le 5$	$R = 31$	$R = 32$	$R = 34$
	$w_{KA\to F} = 109$	$0 \le s_{24} \le 8$		$n = 8$	$m = 7$
			$[5, 21, 5]$	$[3, 21, 0]$	$[5, 21, 1]$
23.	$w_{KA\to MA} = 132$	$0 \le s_9 \le 28$	$\mathcal{F} = 13642$	$\mathcal{F} = 9240$	$\mathcal{F} = 8313$
	$w_{MA\to FD} = 76$	$0 \le s_{11} \le 5$	$R = 23$	$R = 21$	$R = 25$
	$w_{L\to DD} = 48$	$0 \le s_{15} \le 5$		$n = 6$	$m = 8$
	$w_{MA\to DD} = 145$	$0 \le s_{16} \le 5$			
	$w_{KA\to L} = 63$	$0 \le s_{17} \le 8$			
	$w_{KA\to FD} = 93$	$0 \le s_{18} \le 9$			
	$w_{MA\to L} = 73$	$0 \le s_{20} \le 7$	$[5, 13, 5, 5, 8, 5, 0]$	$[5, 10, 5, 5, 0, 5, 0]$	$[5, 7, 5, 0, 7, 0, 0]$
	$w_{KA\to DD} = 82$				
	$w_{FD\to DD} = 98$				
24.	$w_{KA\to MA} = 149$	$0 \le s_9 \le 10$	$\mathcal{F} = 8960$	$\mathcal{F} = 7290$	$\mathcal{F} = 8000$
	$w_{MA\to DD} = 121$	$0 \le s_{15} \le 6$	$R = 21$	$R = 20$	$R = 23$
	$w_{KA\to DD} = 93$	$0 \le s_{24} \le 7$		$n = 4$	$m = 4$
	$w_{KA\to F} = 173$	$0 \le s_{27} \le 5$			
	$w_{F\to DD} = 49$		$[6, 10, 5, 10, 0]$	$[6, 10, 0, 10, 0]$	$[6, 8, 5, 8, 0]$
	$w_{MA\to F} = 68$	$0 \le s_{12} \le 12$			
25.	$w_{KA\to MA} = 112$	$0 \le s_9 \le 28$	$\mathcal{F} = 17888$	$\mathcal{F} = 11808$	$\mathcal{F} = 12872$
	$w_{MA\to DD} = 141$	$0 \le s_{11} \le 9$	$R = 29$	$R = 25$	$R = 32$
	$w_{KA\to DD} = 197$	$0 \le s_{15} \le 5$		$n = 4$	$m = 4$
	$w_{KA\to FD} = 142$	$0 \le s_{16} \le 5$			
	$w_{FD\to DD} = 159$	$0 \le s_{26} \le 11$	$[5, 15, 9, 5, 10]$	$[5, 13, 8, 5, 0]$	$[5, 9, 9, 5, 4]$
	$w_{MA\to FD} = 93$				
26.	$w_{KA\to DD} = 205$	$0 \le s_{12} \le 22$	$\mathcal{F} = 8272$	$\mathcal{F} = 5778$	$\mathcal{F} = 7955$
	$w_{F\to DD} = 105$	$0 \le s_{23} \le 7$	$R = 34$	$R = 30$	$R = 38$
	$w_{KA\to F} = 112$	$0 \le s_{24} \le 6$		$n = 4$	$m = 5$
			$[6, 22, 6]$	$[4, 22, 0]$	$[5, 22, 6]$
27.	$w_{KA\to MA} = 203$	$0 \le s_9 \le 11$	$\mathcal{F} = 6971$	$\mathcal{F} = 5730$	$\mathcal{F} = 4767$
	$w_{MA\to DD} = 95$	$0 \le s_{12} \le 11$	$R = 22$	$R = 24$	$R = 27$
	$w_{KA\to DD} = 82$	$0 \le s_{15} \le 6$		$n = 8$	$m = 8$
	$w_{KA\to F} = 91$	$0 \le s_{24} \le 9$			
	$w_{F-\to DD} = 43$	$0 \le s_{27} \le 7$	$[4, 11, 7, 11, 0]$	$[5, 11, 0, 11, 0]$	$[0, 9, 7, 9, 0]$
	$w_{MA\to F} = 58$				
28.	$w_{KA\to MA} = 221$	$0 \le s_9 \le 18$	$\mathcal{F} = 10516$	$\mathcal{F} = 9505$	$\mathcal{F} = 9352$
	$w_{MA\to DD} = 131$	$0 \le s_{12} \le 23$	$R = 32$	$R = 30$	$R = 35$
	$w_{KA\to DD} = 102$	$0 \le s_{15} \le 8$		$n = 5$	$m = 6$
	$w_{KA\to F} = 53$	$0 \le s_{24} \le 7$			
	$w_{F\to DD} = 59$	$0 \le s_{27} \le 11$	$[8, 13, 11, 13, 0]$	$[8, 17, 11, 6, 0]$	$[8, 10, 11, 10, 0]$
	$w_{MA\to F} = 69$				
29.	$w_{KA\to DD} = 182$	$0 \le s_9 \le 25$	$\mathcal{F} = 6495$	$\mathcal{F} = 5092$	$\mathcal{F} = 5921$
	$w_{MA\to DD} = 105$	$0 \le s_{15} \le 4$	$R = 32$	$R = 35$	$R = 38$
	$w_{KA\to MA} = 89$	$0 \le s_{25} \le 5$		$n = 8$	$m = 8$
			$[4, 25, 3]$	$[2, 25, 0]$	$[4, 25, 1]$
30.	$w_{KA\to MA} = 153$	$0 \le s_9 \le 212$	$\mathcal{F} = 19668$	$\mathcal{F} = 10403$	$\mathcal{F} = 14252$
	$w_{MA\to F} = 85$	$0 \le s_{11} \le 5$	$R = 30$	$R = 26$	$R = 32$
	$w_{L\to DD} = 68$	$0 \le s_{15} \le 6$		$n = 9$	$m = 8$
	$w_{KA\to L} = 122$	$0 \le s_{16} \le 43$			
	$w_{F\to DD} = 57$	$0 \le s_{17} \le 6$			
	$w_{KA\to F} = 68$	$0 \le s_{18} \le 8$			
	$w_{MA\to L} = 142$	$0 \le s_{20} \le 6$	$[6, 19, 5, 13, 6, 8, 5]$	$[6, 11, 5, 6, 0, 6, 0]$	$[6, 13, 5, 7, 6, 7, 0]$
	$w_{KA\to DD} = 59$				
	$w_{MA\to DD} = 79$				

Table 10. Input/Output data regarding routes 21-30.